Decomposition of Drift Vector Field: An application to Multi-machine Transient Stability Enhancement

S. Kulkarni*, M. Parimi, S. Wagh and N. Singh

Abstract—**Knowledge of a potential function is of prime importance in stability analysis of systems. In mechanical systems, energy is considered as potential function. Passivity based methods in literature aim in finding potential function, which may prove unsuccessful for certain class of systems e.g. biological systems. Unfortunately, no general rules exist for the construction of a Lyapunov function, so expertise and intuition of the designer on the specific system is required to define a candidate function. Present paper proposes a systematic method to generate a potential function after decomposing a drift vector field representing a general dynamical system. The method finds application in lossy multi-machine systems for deriving a control law using gradient formulation without solving partial differential equations. The results validated on examples in the area of power system are used in transient stability analysis and enhancement.**

Index Terms—**Absolute stability, Gradient formulation, Hodge decomposition, Lur'e Lyapunov function, Transfer conductances**

I. Introduction

The theory of stability of linear systems is well known in the literature through Nyquist criterion, root locus technique, Routh-Hurwitz criterion, which provide necessary and sufficient conditions for stability. However, for nonlinear systems only sufficiency conditions exist in literature for stability analysis.

A brief overview of various stability criteria developed for nonlinear systems is as follows. Popov criterion gives frequency domain sufficient condition for absolute stability in the form of strict positive realness of a certain transfer function of a system with memoryless, time-invariant nonlinearity lying in first and third quadrant. However, if the nonlinearity is time-varying Popov criterion is not applicable. Hence circle criterion came into existence. Kalman and Yakubwitch established a connection between Popov's frequency domain criterion and the form of Lyapunov function of Lur'e. It was proved that satisfaction of Popov's criterion is a necessary and sufficient condition for existence of Lyapunov function. Kalman showed in [1] that Popov's results correspond to the solvability of original Lur'e equations. Yakubwitch developed frequency domain matrix inequalities representing equivalent algebraic conditions [2]. Popov and Yakubwitch provided further extensions in case of multiple nonlinearities, commonly termed as KYP lemma which is available in different forms. Important feature of KYP lemma is that, it relates the internal (state) stability of a nonlinear system to input-output properties of its subsystems. A linear subsystem satisfies KYP lemma if and only if it is passive or positive system. When the input-output relation is a constant matrix, testing its positive definiteness is possible by simply calculating the eigenvalues and checking whether they are all positive. It is extremely tedious to prove positive realness of a transfer function in case of complex systems such as a power system. Zames in [3], [4] formulated the use of loop transforms to produce operators that satisfy loop gain or passivity condition equivalent to Popov's theory. Use of RL or RC multipliers was introduced to strengthen small gain theorem which directly relates to Popov criterion. Multiplier methods were later generalized for systems with memoryless nonlinearities having sector and slope restrictions. After the work of Popov, a framework incorporating passivity and small gain theorem referred to as dissipative theory was developed by Williems in [5], [6], according to which, combining subsystems that absorb (or dissipate) more energy than they produce (or supply) results in stable (or unstable) system.

Basic concepts of stability emerged from study of equilibrium state of a mechanical system. A close relation between stability and notions of energy, pioneered by A M Lyapunov is elaborated in [7]. The key idea was that if every motion of a system has the property that its energy decreases with time, the system must come to rest irrespective of its initial state. To make the argument more rigorous, Lyapunov insisted that the energy measure $V(x(t))$ of a motion $x(t)$ should be proper, i.e : $V(0) = 0$, $V(x) > 0$, $\forall\, x \neq 0$. The requirement that the V should be decreasing along all trajectories of the system

$$\dot{x} = f(x) \tag{1}$$

takes the form $\dot{V}(x) = \frac{\partial V(x)}{\partial x} f(x) < 0, \forall x \neq 0$, where $\frac{\partial V(x)}{\partial x}$ is the gradient of V along x. If the inner product of this gradient and a tangent vector $\dot{x}$ is constantly negative then surface of $V(x)$ is monotonically decreasing to zero. Together, these conditions are well known conditions for Lyapunov stability, and a function $V(x)$ that satisfies the two inequalities is called the Lyapunov function for the system. Unfortunately, no general rules exist for the construction of a Lyapunov function, so expertise and intuition of the designer on the specific system is practically required to define a candidate function. However, methods addressed in [8] such as first integrals, method of quadratic forms, solutions of Partial Differential Equations (PDE) etc to some extent help in constructing a Lyapunov function.

In view of this, present paper proposes a systematic method to construct an appropriate storage function- a Lur'e-Lyapunov function for a general dynamical system. Major contributions

S. Kulkarni*, M. Parimi, Research scholars, EE, VJTI, Mumbai. samikul63@gmail.com
S. Wagh, faculty, EE, VJTI, and visiting research scholar, TUFTS, N. Singh, Professor, EE, VJTI, Mumbai University, India

of this method are:
$i)$ Systematic decomposition of a general dynamical system (a drift vector field) into its natural constituents and applying it to the pervasive problem of transient stability of lossy n-machine systems.
$ii)$ Computation of structure matrix by solving Linear Matrix Inequalities (LMIs).
$iii)$ Gradient formulation of the exact part of vector field to generate a potential function rather than assuming it and taking care of non-exact part by adding dissipation to the system.
$iv)$ Formulation of generalized control law without solving PDEs.

Organization of the paper is as follows. An overview of different stability criteria and stability in the sense of Lyapunov is discussed in Section I. The decomposition of a drift vector field into its natural components and their significance is described in Section II. After defining a Lur'e problem, Section III gives a systematic method to generate a potential function and a control law for a general nonlinear dynamical system. Power system dynamics being amenable to Lur'e formulation, the proposed method is validated on lossy 2-machine system, in Section IV followed by a generalization of the proposed method for an n-machine lossy power system described in Section V. Finally conclusions and future scope of the method is discussed in Section VI.

II. Decomposition of a vector field

Consider a general nonlinear dynamical system:

$$\dot{x} = f(x) + g(x)u \tag{2}$$

where $f(x)$ and $g(x)$ are smooth vector fields on $\mathcal{X} \in R^n$ which is the operating region of the system. If $u \equiv 0$ is an admissible control resulting in trajectories generated by vector field f. Hence f is called as drift vector field and g is called as control vector field. There exists an energy storage function $\mathcal{H} : \mathcal{X} \to R^+$ which may be zero outside of $\mathcal{X}$. Moreover, $f(x)$ has a natural decomposition, commonly referred to as Hodge decomposition addressed in [9], with respect to storage function $\mathcal{H}$ which is pictorially shown in Fig.1. Mathematically, the drift vector field is represented as:

$$f(x) = \underbrace{f_d(x)}_{Gradient} + \underbrace{f_I(x)}_{Harmonic} + \underbrace{f_{nd}(x)}_{Curl} \quad such \quad that \tag{3}$$

1) $\mathcal{L}_{f_d}\mathcal{H}(x) \leq 0$
2) $\mathcal{L}_{f_{nd}}\mathcal{H}(x)$ is either sign indefinite or non-negative in x
3) $\mathcal{L}_{f_I}\mathcal{H}(x) = 0$

where $\mathcal{L}$ denotes Lie derivative and the vector fields $f_d(x)$(dissipative or exact), $f_{nd}(x)$ (non-dissipative or anti-exact), $f_I(x)$ (invariant) are natural components of $f(x)$ with respect to $\mathcal{H}(x)$ [10]. Rewriting (2) as,

$$\dot{x} = \frac{\partial \mathcal{H}}{\partial x}(f_d(x) + f_I(x) + f_{nd}(x)) + g(x)u \tag{4}$$

Harmonic portion of the vector field contributes in forming the Hamiltonian which is conservative in nature, gradient portion decides stability of the system, for example, if this component is negative definite then the gradient flow is inwards. $f_d(x) + f_I(x)$ constitutes the Lur'e part of the vector field.

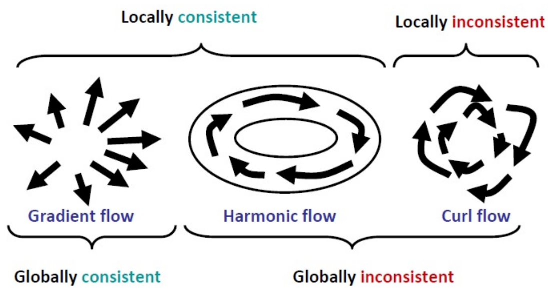

Fig. 1. Natural components of vector field

However, no particular strategy is followed by curl, hence it may be treated as perturbation. There are only two options to overcome the problem of sign-indefinite constituent, either it is absorbed in the skew-symmetric part or it is suppressed by adding dissipation to the system as explained in Section III.

III. Formulation of potential function and control law

A traditional control technique described in [11] normally used to deal with the problem of absolute stability for a certain class of nonlinear systems was proposed by Lur'e, wherein a Nonlinear Isolation Method, is applied to decompose a system into a linear system with a nonlinear memoryless feedback $f(\sigma)$ as shown in Fig.2. Forward path of the system is described as:

$$\dot{x}(t) = Ax(t) + Bu(t), \; y(t) = Cx(t) + Du(t) \tag{5}$$

and feedback subsystem as:

$$z(t) = \sigma[t, y(t)], \; u(t) = -z(t) \tag{6}$$

It is assumed that both forward and feedback subsystems are "square" i.e. both have equal number of inputs and outputs. Given the matrices $A \in R^{n \times n}$, $B \in R^{n \times m}$, $C, D \in R^{m \times m}$, such that pair (A, B) is controllable and pair (C, A) is observable and numbers α and β are such that $\alpha < \beta$, the problem is to derive conditions involving transfer matrix and numbers α, β such that origin is globally, uniformly, asymptotically stable equilibrium point of (5) for every function $\sigma : R^+ \times R^m \to R^m$ belonging to the sector $[\alpha, \beta]$.

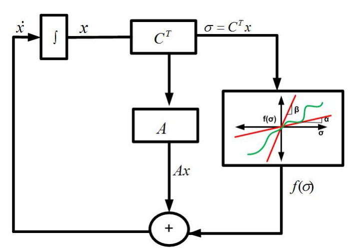

Fig. 2. A system with nonlinearity

Absolute stability problem is commonly referred to as Lur'e problem. Mathematically the Lur'e system is represented as:

$$\dot{x} = Ax - Bf(\sigma), \; \sigma = C'x \tag{7}$$

where, $x \in R^n$, σ is a control variable and $f(\sigma)$ is a nonlinear function such that $\sigma f(\sigma) > 0$ when $\sigma > 0$. Although, form of $f(\sigma)$ in (7) is not specified, it is known to be sector bound and slope restricted as shown in Fig.3. A quadratic inequality is used to bound the nonlinear feedback between two linear

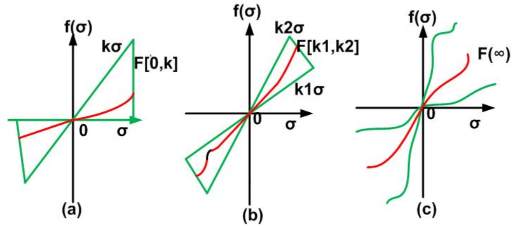

Fig. 3. Different types of nonlinearities

bounds. The sector-bound nonlineariy defined in [12], [13], [14], representing the feedback subsystem of Lur'e system is defined as: suppose $\sigma : R_+ \times R^m \rightarrow R^m$ and the constants $\alpha, \beta \in R$, with $\alpha < \beta$. Then σ is said to belong to the sector $[\alpha, \beta]$ if:

i) $\sigma(t, 0) = 0, \forall t \in R_+$ and

ii) $[\sigma(t, y) - \alpha y]'[\beta y - \sigma(t, y)] \geq 0, \forall t \in R_+, \forall y \in R^m$.

The Lur'e system is said to be absolutely stable,if it has a globally uniformly asymptotically stable equilibrium point at the origin for all nonlinearities in a given sector.

A procedure to generate potential function and to derive a control law is as follows:

1) Given any general dynamical system of the form (2), decompose $f(x)$ into its natural components, to get

$$\dot{x} = \underbrace{f_d(x) + f_I(x)}_{Lur'e} + \underbrace{f_{nd}(x)}_{non-Lur'e} + g(x)u \tag{8}$$

2) The system can be re-written in the generalized gradient form:

$$\dot{x} = -Q\nabla\mathcal{H} + f_{nd}(x) + g(x)u \tag{9}$$

where $\mathcal{H}$ is the Lur'e Lyapunov function obtained by augmenting the Lyapunov energy function with integral of nonlinearity addressed in [15], [16] as:

$$\begin{aligned} \mathcal{H} &= Kinetic\, energy \;+\; Potential\, energy \\ &= x^T P x + \int_0^{C^\top x} f(\sigma) \end{aligned} \tag{10}$$

3) Gradient of $\mathcal{H}$ is:

$$\nabla\mathcal{H} = Px + C^\top f(\sigma) \tag{11}$$

This modifies (9) as:

$$\dot{x} = -QPx - QC^\top f(\sigma) + f_{nd}(x) + g(x)u \tag{12}$$

Comparison of (7) and (12) results into

$$B = QC^\top \quad and \quad A = -QP \tag{13}$$

The structure of Q is known from (13). For n^{th} order system Q is computed by solving LMI:

$$\begin{bmatrix} -(Q+Q^\top) & (B-QC) \\ (B-QC)' & -I_{(n)} \end{bmatrix} \leqslant 0 \tag{14}$$

so that $(Q + Q^\top) \geqslant 0$.
Once Q is computed P is obtained from (13). Thus no assumptions are made while forming structure matrix which is a key contribution of the proposed method. This feature makes it feasible to use the method for n-dimensional system.

4) Splitting the structure matrix Q as:

$$\begin{aligned} Q &= Q_{skew} + Q_{sym} \\ &= (1/2)(Q - Q^\top) + (1/2)(Q + Q^\top) \end{aligned} \tag{15}$$

$\dot{H}$ is computed as:

$$\dot{\mathcal{H}} = -\nabla\mathcal{H}^\top Q_{skew}\nabla\mathcal{H} - \nabla\mathcal{H}^\top Q_{sym}\nabla\mathcal{H} \tag{16}$$

5) Deriving a control law: System is stable if $\dot{\mathcal{H}} \leq 0$. In (16) since the first term is zero, stability of the system is determined by the second term. Control law u_1 is so chosen that it is possible to absorb the actuated part $f_{nd1}(x)$ into the skew symmetric part of the structure matrix. Nonlinearity represented by unactuated part f_{nd2} may be considered as a perturbation in the system and an obvious solution to overcome the perturbation is to add dissipation in the system by modifying the symmetric part of the structure matrix. The closed loop dynamics of the system then becomes:

$$\dot{x} = -\tilde{Q}_{skew}\nabla\mathcal{H} - \tilde{Q}_{sym}\nabla\mathcal{H} \tag{17}$$

where, $\tilde{Q}_{skew}$ is modified Q_{skew} matrix after absorbing $f_{nd1}(x)$ term and $\tilde{Q}_{sym}$ is modified Q_{sym} matrix after adding dissipation. The system is stable if $Q_{sym} \geq 0$. The control law is so designed as to maintain the skewness of Q_{skew} on absorption of terms in f_{nd1}. Mathematically it could be expressed as:
$\tilde{Q}_{skew}(i,j)\nabla\mathcal{H}_j = -\tilde{Q}_{skew}(j,i)\nabla\mathcal{H}_i$ The term absorbed in the location
$Q_{skew}(i,j) = f_{nd1}(x)/(\partial\mathcal{H}/\partial x_j)$
The control law then becomes:
$-\tilde{Q}_{skew}(j,i)\nabla\mathcal{H}_i$

IV. Case study example:A lossy 2-machine system

Consider a lossy 2-machine system represented using flux-decay model given in [17]:

$$\begin{aligned} \dot{\delta}_{12} &= \omega_1 - \omega_2 \\ \dot{\omega}_1 &= -D_1\omega_1 + P_1 - G_{11}E_1^2 - E_1E_2B_{12}\sin(\delta_{12}) \\ &\quad - E_1E_2G_{12}\cos(\delta_{12}) \\ \dot{\omega}_2 &= -D_2\omega_2 + P_2 - G_{22}E_2^2 + E_1E_2B_{12}\sin(\delta_{12}) \\ &\quad - E_1E_2G_{12}\cos(\delta_{12}) \\ \dot{E}_1 &= -a_1E_1 + b_1E_2\cos(\delta_{12} + \alpha_{12}) + E_{f1} + u_1 \\ \dot{E}_2 &= -a_2E_2 + b_2E_1\cos(\delta_{12} - \alpha_{12}) + E_{f2} + u_2. \end{aligned} \tag{18}$$

To decompose $f(x)$ into it's natural components, it is required to modify speed dynamics in (18) as:

$$\begin{aligned} \dot{\omega}_1 &= -D_1\omega_1 - E_{1\star}E_{2\star}B_{12}sin(\delta_{12}) - E_{1\star}E_{2\star}G_{12}cos(\delta_{12}) \\ &\quad - (E_1 - E_{1\star})[E_{2\star}(B_{12}sin(\delta_{12}) + G_{12}cos(\delta_{12})] \\ &\quad - E_1[(E_2 - E_{2\star})(B_{12}sin(\delta_{12}) + G_{12}cos(\delta_{12})] \end{aligned} \tag{19}$$

$$\begin{aligned} \dot{\omega}_2 &= -D_2\omega_2 + E_{1\star}E_{2\star}B_{12}sin(\delta_{12}) - E_{1\star}E_{2\star}G_{12}cos(\delta_{12}) \\ &\quad (E_2 - E_{2\star})[E_1(B_{12}sin(\delta_{12}) + G_{12}cos(\delta_{12})] \\ &\quad + E_{2\star}[(E_1 - E_{1\star})(B_{12}sin(\delta_{12}) - G_{12}cos(\delta_{12})] \end{aligned} \tag{20}$$

The equilibrium point for the system is $(\delta_{12\star}, 0, 0, E_{1\star}, E_{2\star})$, which modifies excitation dynamics in (18) to:

$$\dot{E}_1 = -a_1(E_1 - E_{1\star}) - a_1E_{1\star} + b_1E_2cos(\delta_{12} + \alpha_{12}) + E_{f1} + u_1 \quad (21)$$

$$\dot{E}_2 = -a_2(E_2 - E_{2\star}) - a_2E_{2\star} + b_2E_1cos(\delta_{12} - \alpha_{12}) + E_{f1} + u_2 \quad (22)$$

Applying the necessary condition for the existence of an equilibrium as mentioned in [18] it is observed that $P_1 + P_2 = E_1^2G_{11} + E_2^2G_{22}$. For the system (18) the potential function $\mathcal{H}$ becomes:

$$\mathcal{H} = \frac{1}{2}(\omega_1^2 + \omega_2^2) + \frac{1}{2}\gamma_1(E_1 - E_{1\star})^2 + \frac{1}{2}\gamma_2(E_2 - E_{2\star})^2 + E_{1\star}E_{2\star}B_{12}cos(\delta_{12}) \quad (23)$$

The gradient of which is:

$$[-E_{1\star}E_{2\star}G_{12}sin(\delta_{12})\ \omega_1\ \omega_2\ \gamma_1(E_1 - E_{1\star})\ \gamma_2(E_2 - E_{2\star})]' \quad (24)$$

Decomposing system (18) into natural components as:

$$\begin{bmatrix} \dot{\delta_{12}} \\ \dot{\omega_1} \\ \dot{\omega_2} \\ \dot{E_1} \\ \dot{E_2} \end{bmatrix} = \{-\underbrace{\begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 1 & -D_1 & 0 & 0 & 0 \\ -1 & 0 & -D_2 & 0 & 0 \\ 0 & 0 & 0 & -a_1/r_1 & 0 \\ 0 & 0 & 0 & 0 & -a_2/r_2 \end{bmatrix}}_{f_d + f_I} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \star 1 & \star 2 \\ 0 & 0 & 0 & \star 3 & \star 4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}_{f_{nd1}}\}\nabla\mathcal{H} + \underbrace{\begin{bmatrix} 0 \\ f_{nd21} \\ f_{nd22} \\ 0 \\ 0 \end{bmatrix}}_{f_{nd2}} \quad (25)$$

where, $f_{nd21} = -E_{1\star}E_{2\star}G_{12}cos(\delta_{12}) = f_{nd22}$ and

$$\star 1 = f_{nd11}(2,4) = -\frac{E_{2\star}(B_{12}sin(\delta_{12}) + G_{12}cos(\delta_{12}))}{\gamma_1}$$

$$\star 2 = f_{nd12}(2,5) = -\frac{(E_1(B_{12}sin(\delta_{12}) + G_{12}cos(\delta_{12}))}{\gamma_2}$$

$$\star 3 = f_{nd13}(3,4) = \frac{(E_{2\star}(B_{12}sin(\delta_{12}) - G_{12}cos(\delta_{12}))}{\gamma_1}$$

$$\star 4 = f_{nd14}(3,5) = -\frac{E_1(B_{12}sin(\delta_{12}) + G_{12}cos(\delta_{12}))}{\gamma_2} \quad (26)$$

First term on RHS of (25) is the structure matrix Q which is split into Q_{sym} i.e. dissipative component, and Q_{skew} i.e. invariant component. Actuated part f_{nd1} i.e portion of sign indefinite component represented in terms of $\nabla\mathcal{H}$ could be absorbed in Q_{skew}. However, it is not possible to absorb a portion of sign-indefinite component viz. f_{nd2} which is due to non-trivial transfer conductances. The only solution to overcome this problem is to suppress its effect by adding dissipation in the system, through the tuning parameters γ_1 and γ_2, by deriving a suitable control law. For the system (18) the control laws are:

$$u_1 = a_1E_{1\star} - b_1E_2cos(\delta_{12} + \alpha_{12}) - E_{f1} - f_{nd11}\omega_1 - f_{nd12}\omega_2 - r_1\gamma_1(E_1 - E_{1\star}) \quad (27)$$

$$u_2 = a_2E_{2\star} - b_2E_1cos(\delta_{12} - \alpha_{12}) - E_{f2} - f_{nd13}\omega_1 - f_{nd14}\omega_2 - r_2\gamma_2(E_2 - E_{2\star}) \quad (28)$$

The proposed method of decomposition of $f(x)$ into natural constituents may be generalized to an n-machine system.

V. Generalization for lossy n-machine system

Consider an n-machine system represented by flux decay model:

$$\dot{\delta}_{ij} = \omega_i - \omega_j\ for\ i = 1\ to\ n$$

$$\dot{\omega}_i = -D_i\omega_i + P_i - G_{ii}E_i^2 - \sum_{j=1,j\neq i}^{j=n}[E_iE_jB_{ij}\sin(\delta_{ij}) - E_iE_jG_{ij}\cos(\delta_{ij})]$$

$$\dot{E}_i = -a_iE_i + \sum_{j=1,jneqi}^{j=n} b_iE_j\cos(\delta_{ij} + \alpha_{1j}) + E_{fi} + u_i \quad (29)$$

where $\delta_{ij} = -\delta_{ji}$ and $\alpha_{ij} = tan^{-1}\frac{G_{ij}}{B_{ij}}$ Also for $j > i$, $\delta_{ij} = \delta_{1j} - \delta_{1i}$ A necessary condition for the existence of an equilibrium as mentioned in [18] is :

$$\sum_{i=1}^{n} P_i = \sum_{i=1}^{n} G_{ii}E_i^2 \quad (30)$$

The state vector x for n-machine system is:

$$[\delta_{12}\ \delta_{13}\ \cdots\ \delta_{1n}\ \omega_1\ \omega_2\ \cdots\ \omega_n\ E_1\ E_2\ \cdots\ E_n]' \quad (31)$$

Modifying frequency dynamics of (29),

$$\dot{\omega}_i = -D_i\omega_i + \sum_{j=1,j\neq i}^{j=n} E_{i\star}E_{j\star}B_{ij}\sin(\delta_{ij}) - (E_i - E_{i\star})[\sum_{j=1,j\neq i}^{j=n} E_{j\star}(B_{ij}sin(\delta_{ij}) + G_{ij}cos(\delta_{ij})] - E_i[\sum_{j=1,j\neq i}^{j=n}(E_j - E_{j\star})(B_{ij}sin(\delta_{ij}) + G_{ij}cos(\delta_{ij}))] - \sum_{j=1,j\neq i}^{j=n} E_{i\star}E_{j\star}G_{ij}cos(\delta_{ij}) \quad (32)$$

The equilibrium point for the system is

$$(\delta_{12\star},\ \delta_{13\star},\ ..,\ \delta_{1n\star}, 0,\ 0,\ ..,\ 0\ E_{1\star},\ E_{2\star},\ ..,\ E_{n\star}) \quad (33)$$

Modifying excitation dynamics in (29) as:

$$\dot{E}_i = -a_i(E_i - E_{i\star}) - a_iE_{i\star} + \sum_{i=1,i\neq j}^{i=n} b_iE_jcos(\delta_{ij} + \alpha_{ij}) + E_{fi} + u_i \quad (34)$$

Applying the procedure described in Section III, the potential function is:

$$\mathcal{H} = \frac{1}{2}\sum_{i=1,i\neq j}^{i=n}\omega_i^2 + \frac{1}{2}\sum_{i=1,i\neq j}^{i=n}\gamma_i(E_i - E_{i\star})^2 - \sum_{j=2}^{j=n} E_{1\star}E_{j\star}B_{1j}sin(\delta_{1j}) \quad (35)$$

The Hodge decomposition of $f(x)$ in matrix form becomes:

$$\begin{bmatrix}\dot{\delta}_{12}\\ \vdots\\ \dot{\delta}_{1n}\\ \dot{\omega}_1\\ \vdots\\ \dot{\omega}_n\\ \dot{E}_1\\ \vdots\\ \dot{E}_n\end{bmatrix} = \{-\underbrace{\begin{bmatrix} \underbrace{[0]}_{(n-1,n-1)} & [\ \underbrace{[-1]}_{(n-1,1)}\ \underbrace{[I]}_{(n-1,n-1)}\ ] & \underbrace{[0]}_{(n-1,n)} \\ \begin{bmatrix}\underbrace{[1]}_{(1,n-1)}\\ -\underbrace{[I]}_{(n,n-1)}\end{bmatrix} & \underbrace{[Diag(D_i)]}_{(n,n)} & \underbrace{[0]}_{(n,n)} \\ \underbrace{[0]}_{(n,n-1)} & \underbrace{[0]}_{(n,n)} & \underbrace{[Diag(\frac{a_i}{r_i})]}_{(n,n)}\end{bmatrix}}_{Q=f_d+f_I}$$

$$+\underbrace{\begin{bmatrix}\underbrace{[0]}_{(2n-1,n)} & \underbrace{[0]}_{(n,n)}\\ \underbrace{[0]}_{(2n-1,n-1)} & \underbrace{[f_{nd1}]}_{(n,n)}\\ \underbrace{[0]}_{(2n-1,n)} & \underbrace{[0]}_{(n,n)}\end{bmatrix}}_{f_{nd1}}\}\nabla\mathcal{H} + \underbrace{\begin{bmatrix}\underbrace{[0]}_{(n-1,1)}\\ \underbrace{[f_{nd2}]_{(n,1)}}\\ \underbrace{[0]}_{(n,1)}\end{bmatrix}}_{f_{nd2}} \tag{36}$$

The diagonal elements if matrix $[f_{nd1}]$ are:

$$-(E_i - E_{i\star})[\sum_{j=1,j\neq i}^{j=n} E_{j\star}(B_{ij}sin(\delta_{ij}) + G_{ij}cos(\delta_{ij})] \tag{37}$$

and off-diagonal elements are: $-E_i[\sum_{j=1,j\neq i}^{j=n}(E_j - E_{j\star})(B_{ij}sin(\delta_{ij}) + G_{ij}cos(\delta_{ij}))]$ extracted from (32). Similarly, $[f_{nd2}]$ is comprised of terms in (32): $-\sum_{i=1,i\neq j}^{i=n} E_{i\star}E_{j\star}G_{ij}cos(\delta_{ij})$. As mentioned earlier in (16), stability is ensured if $\dot{\mathcal{H}} \leq 0$. The nonlinearities f_{nd1} in the equations representing speed dynamics are absorbed in Q_{skew} of dimension $(3n-1, 3n-1)$, for an n-machine system. The entries of f_{nd1} has a dimension of (ii, kk) where $ii = (n\,to\,2n-1)$ and $kk = (2n\,to\,3n-1)$. However, this disturbs the skewness of Q_{skew}. In order to maintain skewness (kk, ii) elements of the matrix are filled by identical entries with opposite signs which modifies Q_{skew} to $\tilde{Q}_{skew}$. since $\nabla\mathcal{H}^\top Q_{skew}\nabla\mathcal{H} = 0$, to assure stability of the system $\nabla\mathcal{H}^\top Q_{sym}\nabla\mathcal{H} \leq 0$. This is possible only by adding dissipation in the system through symmetric part of interconnection matrix. The control laws appearing in the excitation dynamics are:

$$u_i = a_iEi - \sum_{i=1,i\neq j}^{i=n} b_iE_j\cos(\delta_{ij} + \alpha_{1j}) - E_{fi} - \sum_{i=1}^{i=n} f_{nd1i}\omega_i - r_i\gamma_i(E_i - E_{i\star}) \tag{38}$$

where r_i is a tuning parameter to be adjusted so as to make $\dot{\mathcal{H}} = 0$. In the control approach explained, it is to be noted that the effect of unactuated factor may increase with the size of the system. Such a case could be stabilized with the help of a global controller such as Thyristor Controlled Series Compensator (TCSC) using edge weight control method proposed in [19].

VI. Conclusions and future scope

The drift vector field decomposition method gives a generic perspective to analyze a wide range of applications of nonlinear dynamical systems. The approach highlights the manipulations in interconnection matrix to derive a suitable control law. However, the method may be explored to the class of systems wherein different potential function could be formulated to assure stability. The method has efficiently addressed the long-lasting problem of finding a control law for lossy n-machine systems.

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